A NOTE ON SOME GENERALIZED CLOSURE AND INTERIOR OPERATORS IN A TOPOLOGICAL SPACE

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Abstract. If $X$ is a topological space and $A \subseteq X$, then the number of distinct sets that can be obtained from $A$ by using all possible compositions for operators $i_\gamma$, $c_\gamma$ (where $\gamma = \sigma, \pi, \alpha, \beta$) introduced by Császár is at the most 25. Explicit expressions for these sets are provided. An example is provided where all the 25 different sets are determined. The result is also discussed for special cases such as when the space is extremally disconnected, resolvable, open-unresolvable, and partition spaces.

1. Introduction

Kuratowski’s closure complement theorem (also known as 14 set theorem) [11] has been a guiding source of research not only in topology, but also in various other fields such as Approximation Theory, Relational algebra, Formal Language, Computer programming [4, 5, 9, 15], etc. Peleg [15], while investigating the transitive closure of a binary relation, came across several closure operators which do not satisfy some of the four of Kuratowski’s closure axioms, though their properties suffice to maintain “closure complement phenomenon”. Similar kind of generalized closure operators are generated by the monotonic mappings introduced by Á. Császár [6]. When several such operators are considered simultaneously and composed, the study of closure complement phenomenon becomes complicated and highly interesting. In the present note, we provide our investigations regarding all possible compositions of four such generalized closure operators and their corresponding interior operators.

Let $X$ be a non empty set and let $\gamma : \mathcal{P}(X) \rightarrow \mathcal{P}(X)$ satisfy $\gamma(A) \subseteq \gamma(B)$ for $A \subseteq B$, where $\mathcal{P}(X)$ is the power set of $X$. According to Á. Császár [6], $A \subseteq X$ is called $\gamma$-open if $A \subseteq \gamma(A)$. The $\gamma$-open sets of $X$ form a generalized topology [8] (GT in brief) on $X$ in the sense that (i) $\phi$ is $\gamma$-open and (ii) any union of $\gamma$-open sets is again $\gamma$-open. A set is called $\gamma$-closed if its complement is $\gamma$-open. For $A \subseteq X$, the largest $\gamma$-open set contained in $A$ is called the $\gamma$-interior of $A$ and is denoted by $i_\gamma(A)$. Similarly, the smallest $\gamma$-closed set containing $A$ is called the $\gamma$-closure of $A$ and is denoted by $c_\gamma(A)$.

If $X$ is a topological space and $i$ and $c$ denote the interior and closure operators respectively, then $\gamma = ci, ici, ic$ and $cic$ give rise to important families of $\gamma$-open sets. The corresponding $\gamma$-open sets are known as semi-open, $\alpha$-open, $\pi$-open

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and \( \beta \)-open sets respectively, in the literature. In \cite{7}, Á. Császár has shown that in some cases (including the four classical cases mentioned above), the \( \gamma \)-interior \( i_{\gamma}(A) \) and the \( \gamma \)-closure \( c_{\gamma}(A) \) of \( A \) are easily obtained by explicit formulas. In the present note, we provide explicit formulas arising out of all possible compositions of different \( i_{\gamma} \)'s and \( c_{\gamma} \)'s, where \( \gamma = ci, ic, ic, \) or \( cic \) in the topological framework. We have found that for \( A \subseteq X \), from the possible compositions of \( i_{\alpha} \), \( i_{\sigma} \), \( i_{\pi} \), \( i_{\beta} \), \( c_{\alpha} \), \( c_{\sigma} \), \( c_{\pi} \) and \( c_{\beta} \), we get at most 25 different sets. We have provided explicit expressions of all these sets. We have also provided an example where this bound is achieved. If the topology satisfies some extra properties such as in an extremally disconnected space, \( OU \)-space and Partition space etc. the upper bound is less than 25. We have provided a discussion on such particular cases in the paper. Unlike in Kuratowski’s 14 set theorem, these 25 operators do not form a monoid, but, rather, a semi-group. Algebraic properties of each of \( i_{\gamma} \), \( c_{\gamma} \) are investigated in our paper \cite{17}.

2. THE MAIN RESULT

First of all, we provide some definitions and notations:

**Definition 2.1.** Let \((X, \tau)\) be a topological space and \( A \subseteq X \). Then, \( A \) is called

(i) **semi-open** \cite{12} if \( A \subseteq cl(int(A)) \);
(ii) **\( \alpha \)-open** \cite{14} if \( A \subseteq int(cl(int(A))) \);
(iii) **pre-open** \cite{13} if \( A \subseteq int(cl(A)) \);
(iv) **\( \beta \)-open** \cite{1} if \( A \subseteq cl(int(cl(A))) \).

The complement of a **semi-open** set is called **semi-closed**. The largest semi-open set contained in \( A \) is denoted by \( i_{\sigma}(A) \). The smallest semi-closed set containing \( A \) is denoted by \( c_{\sigma}(A) \). Thus, the concept of a semi-open set gives rise to the operators \( c_{\sigma} \) and \( i_{\sigma} \). In an analogous way, the concept of an \( \alpha \)-open set gives rise to the operators \( c_{\alpha} \) and \( i_{\alpha} \), the concept of a pre-open set gives rise to the operators \( c_{\pi} \) and \( i_{\pi} \), and the concept of a \( \beta \)-open set gives rise to the operators \( c_{\beta} \) and \( i_{\beta} \).

**Theorem 2.2.** \cite{7} In a topological space \((X, \tau)\) with \( A \subseteq X \), we have,

(i) \( c_{\sigma}(A) = A \cup int(cl(A)) \);
(ii) \( i_{\sigma}(A) = A \cap cl(int(A)) \);
(iii) \( c_{\alpha}(A) = A \cup cl(int(cl(A))) \);
(iv) \( i_{\alpha}(A) = A \cap int(cl(int(A))) \);
(v) \( c_{\pi}(A) = A \cup cl(int(A)) \);
(vi) \( i_{\pi}(A) = A \cap int(cl(A)) \);
(vii) \( c_{\beta}(A) = A \cup int(cl(int(A))) \);
(viii) \( i_{\beta}(A) = A \cap cl(int(cl(A))) \).

The following result will be used to prove the main theorem:

**Lemma 2.3.** In a topological space \((X, \tau)\) with \( A \subseteq X \), the following hold:

\[
cl\{A \cap int(cl(int(A)))\} = cl(int(A));
\]
\[
int\{A \cup cl(int(cl(A)))\} = int(cl(A));
\]
NOTE ON GENERALIZED CLOSURE AND INTERIOR OPERATORS

13

\[
\begin{align*}
\text{cl}[A \cap \text{cl}(\text{int}(A))] &= \text{cl}(\text{int}(A)); \\
\text{int}[A \cup \text{int}(\text{cl}(A))] &= \text{int}(\text{cl}(A)); \\
\text{cl}[A \cap \text{int}(\text{cl}(A))] &= \text{cl}(\text{int}(\text{cl}(A))); \\
\text{int}[A \cup \text{cl}(\text{int}(A))] &= \text{int}(\text{cl}(\text{int}(A))); \\
\text{cl}[A \cap \text{cl}(\text{int}(\text{cl}(A)))) &= \text{cl}(\text{int}(\text{cl}(A))); \\
\text{int}[A \cup \text{int}(\text{cl}(\text{int}(A))) &= \text{int}(\text{cl}(\text{int}(A))); \\
\text{cl}[A \cap \text{cl}(\text{int}(A)) \cap \text{int}(\text{cl}(\text{int}(A))) &= \text{cl}(\text{int}(\text{cl}(A))); \\
\text{int}[A \cup \text{int}(\text{cl}(\text{int}(A))) \cup \text{int}(\text{cl}(\text{int}(A))) &= \text{int}(\text{cl}(\text{int}(A))); \\
\text{cl}[\text{cl}(\text{int}(A)) \cap \text{int}(\text{cl}(\text{int}(A))) &= \text{cl}(\text{int}(\text{cl}(A))); \\
\text{int}[\text{cl}(\text{int}(A)) \cup \text{int}(\text{cl}(\text{int}(A))) &= \text{int}(\text{cl}(\text{int}(A))).
\end{align*}
\]

Proof. Let \((X, \tau)\) be a topological space and \(A \subseteq X\). Then, \(\text{cl}[A \cap \text{int}(\text{cl}(\text{int}(A))) \subseteq \text{cl}(A) \cap \text{cl}(\text{int}(\text{cl}(\text{int}(A)))) \subseteq \text{cl}(A) \cap \text{cl}(\text{int}(A)) = \text{cl}(\text{int}(A))\). Again, consider \(\text{int}(A) \subseteq A\) and \(\text{int}(A) \subseteq \text{int}(\text{cl}(\text{int}(A)))\), thus \(\text{int}(A) \subseteq A \cap \text{int}(\text{cl}(\text{int}(A)))\). Hence, \(\text{cl}(\text{int}(A)) \subseteq \text{cl}[A \cap \text{int}(\text{cl}(\text{int}(A)))\). Therefore, \(\text{cl}[A \cap \text{int}(\text{cl}(\text{int}(A))) = \text{cl}(\text{int}(A)).\) One can prove all the other equalities in an analogous way. \(\square\)

Now we come to the main result:

**Theorem 2.4.** Let \((X, \tau)\) be a topological space and \(A \subseteq X\). Then, the total number of distinct sets that can be obtained from \(A\) by repeatedly using the operators \(i_\gamma, c_\gamma\) (where \(\gamma = \sigma, \pi, \alpha, \beta\)) is at most 25 and there exists a topological space \((X, \tau)\) and a set \(A \subseteq X\) from which all these 25 sets can be realized.

Proof. In a topological space \(X\) with \(A \subseteq X\), we have

\[
\begin{align*}
\text{i}_\sigma \circ \text{i}_\sigma (A) &= \text{i}_\sigma (\text{i}_\sigma (A)) = \text{i}_\sigma (A \cap \text{cl}(\text{int}(A))) \\
&= (A \cap \text{cl}(\text{int}(A))) \cap \text{cl}(A \cap \text{cl}(\text{int}(A))) \\
&= (A \cap \text{cl}(\text{int}(A))) \cap \text{cl}(A \cap \text{cl}(\text{int}(A))) \\
&= A \cap \text{cl}(\text{int}(A)) \\
&= \text{i}_\sigma (A).
\end{align*}
\]

We express this by \(\text{i}_\sigma \circ \text{i}_\sigma = \text{i}_\sigma\). In the case of \(\text{i}_\sigma\) and \(\text{i}_\pi\), we have

\[
\begin{align*}
\text{i}_\sigma \circ \text{i}_\pi (A) &= \text{i}_\sigma (A \cap \text{int}(\text{cl}(A))) \\
&= (A \cap \text{int}(\text{cl}(A))) \cap \text{cl}(A \cap \text{int}(\text{cl}(A))) \\
&= (A \cap \text{int}(\text{cl}(A))) \cap \text{cl}(A \cap \text{int}(\text{cl}(A))) \\
&= A \cap \text{cl}(\text{int}(A)) \cap \text{int}(\text{cl}(A)).
\end{align*}
\]

We express this by \(\text{i}_\sigma \circ \text{i}_\pi = A \cap \text{int}(\text{cl}(A)) \cap \text{cl}(\text{int}(A)).\)

We put together similar results by using the following composition tables. For convenience, we write \(c\) and \(i\) for \(\text{cl}\) and \(\text{int}\) respectively.

From Tables 1–4, it is clear that so far 21 distinct sets have been realized from a given set \(A\) by the compositions listed in the tables. We enumerate them as
follows. For our convenience we avoid writing $A$, that is, $i_\sigma(A)$ is written as simply $i_\sigma$ and so on.

1 := ici, \hspace{1cm} 2 := ci, \hspace{1cm} 3 := ic, \\
4 := cic, \hspace{1cm} 5 := i_\alpha, \hspace{1cm} 6 := i_\sigma, \\
7 := i_\pi, \hspace{1cm} 8 := i_\beta, \hspace{1cm} 9 := c_\alpha, \\
10 := c_\pi, \hspace{1cm} 11 := c_\sigma, \hspace{1cm} 12 := c_\beta, \\
13 := i_\sigma i_\pi = A \cap ic(A) \cap ci(A), \hspace{1cm} 14 := i_\sigma c_\alpha = [A \cap cic(A)] \cup ic(A),
In the final exhaustive table, we show that these 25 sets are all the possible sets that can be obtained by using the operators \( i_\alpha, i_\sigma, i_\pi, i_\beta, c_\alpha, c_\sigma, c_\pi \) and \( c_\beta \). For our convenience, we use only the numerals to represent a set in the table. For example, 1, 2 and 3 represent \( \text{ic}(A), \text{ci}(A) \) and \( \text{ic}(A) \), respectively as they have been listed above. Verification of the calculations involved in preparing the composition table is not much difficult (it is based on Lemma 2.3, the above tables and the Kuratowski’s closure-complement theorem) and is left to the reader. The authors have also verified the associativity of the compositions using “Light’s associativity test” [3].

### Table 5

| \( \sigma \) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
| 15 | \( i_\sigma c_\beta = [A \cap \text{ci}(A)] \cup \text{ic}(A), \) | 16 | \( i_\pi c_\sigma = [A \cup \text{ci}(A)] \cap \text{ic}(A), \) |
| 17 | \( i_\pi c_\beta = [A \cap \text{ic}(A)] \cup \text{ic}(A), \) | 18 | \( i_\beta c_\pi = [A \cap \text{cic}(A)] \cup \text{ci}(A), \) |
| 19 | \( i_\beta c_\beta = [A \cap \text{cic}(A)] \cup \text{ic}(A), \) | 20 | \( c_\sigma c_\pi = A \cup \text{ci}(A) \cup \text{ic}(A), \) |
| 21 | \( c_\pi i_\pi = [A \cap \text{ic}(A)] \cup \text{ic}(A). \) |
Below we provide an example of a topological space, where the bound of 25 sets has been demonstrated.

**Example 2.5.** Let $X = \mathbb{R}$ be the set of real numbers with the usual topology. Let $A \subseteq X$ be defined by

$$A = \{-1/n, \ n \in \mathbb{N}\} \cup \left[1, 3\right] \setminus \{2 + 1/n, n \in \mathbb{N}\} \cup \left[(5, 7) \cap \left(\mathbb{Q} \cup \bigcup_{n=1}^{\infty} \left(6 + \frac{1}{2n\pi}, 6 + \frac{1}{(2n-1)\pi}\right)\right]\right] \cup (-3, -2).$$

Then, we have

(i) $\text{ic}(A) = (1, 3) \cup (5, 7) \cup (-3, -2);$ 

(ii) $\text{cic}(A) = [1, 3] \cup [5, 7] \cup [-3, -2];$

(iii) $\text{ci}(A) = [1, 3] \cup \{6\} \cup \bigcup_{n=1}^{\infty} \left(6 + \frac{1}{2n\pi}, 6 + \frac{1}{(2n-1)\pi}\right) \cup [-3, -2];$

(iv) $\text{ici}(A) = (1, 3) \cup \bigcup_{n=1}^{\infty} \left(6 + \frac{1}{2n\pi}, 6 + \frac{1}{(2n-1)\pi}\right) \cup (-3, -2);$ 

(v) $A \cap \text{ic}(A) = (1, 3) \setminus \{2 + 1/n, \ n \in \mathbb{N}\}$ 

$$\cup \left[(5, 7) \cap \left(\mathbb{Q} \cup \bigcup_{n=1}^{\infty} \left(6 + \frac{1}{2n\pi}, 6 + \frac{1}{(2n-1)\pi}\right)\right]\right] \cup (-3, -2);$$

(vi) $A \cup \text{ic}(A) = \{-1/n, n \in \mathbb{N}\} \cup [1, 3] \cup (5, 7) \cup (-3, -2);$

(vii) $A \cap \text{cic}(A) = [1, 3] \setminus \{2 + 1/n, \ n \in \mathbb{N}\}$ 

$$\cup \left[(5, 7) \cap \left(\mathbb{Q} \cup \bigcup_{n=1}^{\infty} \left(6 + \frac{1}{2n\pi}, 6 + \frac{1}{(2n-1)\pi}\right)\right]\right] \cup (-3, -2);$$

(viii) $A \cup \text{cic}(A) = \{-1/n, n \in \mathbb{N}\} \cup [1, 3] \cup [5, 7] \cup [-3, -2];$

(ix) $A \cap \text{ci}(A) = [1, 3] \setminus \{2 + 1/n, \ n \in \mathbb{N}\} \cup \{6\}$ 

$$\cup \bigcup_{n=1}^{\infty} \left(6 + \frac{1}{2n\pi}, 6 + \frac{1}{(2n-1)\pi}\right) \cup (-3, -2);$$
(x) 
\[ A \cup \text{ci}(A) = \{-1/n, n \in \mathbb{N}\} \cup [1, 3] \]
\[ \cup \left( (5, 7] \cap \left( \bigcup_{n=1}^{\infty} \left[ 6 + \frac{1}{2n\pi}, 6 + \frac{1}{(2n-1)\pi} \right] \right) \right) \cup [-3, -2]; \]

(xi) 
\[ A \cap \text{ici}(A) = (1, 3) \setminus \{2 + 1/n, n \in \mathbb{N}\} \cup \bigcup_{n=1}^{\infty} \left( 6 + \frac{1}{2n\pi}, 6 + \frac{1}{(2n-1)\pi} \right) \cup (-3, -2); \]

(xii) 
\[ A \cup \text{ici}(A) = \{-1/n, n \in \mathbb{N}\} \cup [1, 3] \]
\[ \cup \left( (5, 7] \cap \left( \bigcup_{n=1}^{\infty} \left[ 6 + \frac{1}{2n\pi}, 6 + \frac{1}{(2n-1)\pi} \right] \right) \right) \cup (-3, -2); \]

(xiii) 
\[ \text{ic}(A) \cap \text{ci}(A) = (1, 3) \cup \{6\} \cup \bigcup_{n=1}^{\infty} \left( 6 + \frac{1}{2n\pi}, 6 + \frac{1}{(2n-1)\pi} \right) \cup (-3, -2); \]

(xiv) 
\[ \text{ic}(A) \cup \text{ci}(A) = [1, 3] \cup (5, 7) \cup (-3, -2); \]

(xv) 
\[ A \cap \text{ic}(A) \cap \text{ci}(A) = (1, 3) \setminus \{2 + 1/n, n \in \mathbb{N}\} \cup \{6\} \]
\[ \cup \bigcup_{n=1}^{\infty} \left( 6 + \frac{1}{2n\pi}, 6 + \frac{1}{(2n-1)\pi} \right) \cup (-3, -2); \]

(xvi) 
\[ A \cup \text{ic}(A) \cup \text{ci}(A) = \{-1/n, n \in \mathbb{N}\} \cup [1, 3] \cup (5, 7) \cup [-3, -2]; \]

(xvii) 
\[ [A \cap \text{ic}(A)] \cup \text{ci}(A) = [1, 3] \cup (5, 7) \cup (-3, -2); \]

(xviii) 
\[ [A \cap \text{ci}(A)] \cup \text{ici}(A) = [1, 3] \cup \{6\} \cup \bigcup_{n=1}^{\infty} \left( 6 + \frac{1}{2n\pi}, 6 + \frac{1}{(2n-1)\pi} \right) \cup (-3, -2); \]

(xix) 
\[ [A \cup \text{ci}(A)] \cap \text{ci}(A) = (1, 3) \]
\[ \cup \left( (5, 7] \cap \left( \bigcup_{n=1}^{\infty} \left( 6 + \frac{1}{2n\pi}, 6 + \frac{1}{(2n-1)\pi} \right) \right) \right) \cup (-3, -2); \]

(xx) 
\[ [A \cap \text{ci}(A)] \cup \text{ici}(A) = (1, 3) \]
\[ \cup \left( (5, 7] \cap \left( \bigcup_{n=1}^{\infty} \left( 6 + \frac{1}{2n\pi}, 6 + \frac{1}{(2n-1)\pi} \right) \right) \right) \cup (-3, -2); \]
A. GUPTA and R. D. SARMA

\[ [A \cap \text{cic}(A)] \cup \text{ci}(A) = [1, 3] \]
\[ \cup \left[ (5, 7] \cap \left( \mathbb{Q} \cup \bigcup_{n=1}^{\infty} \left( 6 + \frac{1}{2n\pi}, 6 + \frac{1}{(2n-1)\pi} \right) \right) \right] \cup [-3, -2]; \]

\[ [A \cap \text{ic}(A)] \cup \text{i}(A) = [1, 3] \]
\[ \cup \left[ (5, 7] \cap \left( \mathbb{Q} \cup \bigcup_{n=1}^{\infty} \left( 6 + \frac{1}{2n\pi}, 6 + \frac{1}{(2n-1)\pi} \right) \right) \right] \cup [-3, -2]; \]

\[ [A \cap \text{ic}(A)] \cup \text{ci}(A) = [1, 3] \]
\[ \cup \left[ (5, 7] \cap \left( \mathbb{Q} \cup \bigcup_{n=1}^{\infty} \left( 6 + \frac{1}{2n\pi}, 6 + \frac{1}{(2n-1)\pi} \right) \right) \right] \cup [-3, -2]; \]

\[ [A \cup \text{ci}(A) \cup \text{ic}(A)] \cap \text{ci}(A) = [1, 3] \cup (5, 7] \cup [-3, -2]; \]

\[ [A \cap \text{ci}(A) \cap \text{ic}(A)] \cup \text{i}(A) = (1, 3) \cup \{6\} \]
\[ \cup \bigcup_{n=1}^{\infty} \left( 6 + \frac{1}{2n\pi}, 6 + \frac{1}{(2n-1)\pi} \right) \cup (-3, -2). \]

3. SOME SPECIAL CASES

If the topology on \( X \) satisfies some extra conditions, then the upper bound obtained above may be less than 25. We have the following results.

**Definition 3.1.** A topological space \((X, \tau)\) is said to be

(i) **extremally disconnected** [16] if the closure of any open set is open,

(ii) **resolvable** [10] if it contains a dense set with empty interior,

(iii) **open unresolvable** [2] if no open subspace is resolvable,

(iv) **partition space** [18] if its open sets form a Boolean algebra.

For open unresolvable space, we use the notation \( \text{OU-space} \).

**Corollary 3.2.** In the case of \((X, \tau)\) being extremally disconnected (\( \text{OU-space, partition space, extremally disconnected OU-space or discrete space} \)), the upper bound obtained above reduces to 7 (resp. 7, 3, 3 or 1).

**Case 1.** Let \((X, \tau)\) be extremally disconnected and \( A \subseteq X \). Then \( \text{ci}(A) \) is an open set. Hence, \( \text{i}(A) = \text{ci}(A) \) and \( \text{cic}(A) = \text{ic}(A) \). Thus the distinct sets obtained from the operators \( i_{\gamma}, c_{\gamma} \) (where \( \gamma = \alpha, \pi, \sigma, \beta \)) are \( \text{i}(A) = \text{ci}(A) \), \( \text{ic} = \text{cic} \), \( i_{\alpha} = i_{\pi} = i_{\beta}, c_{\sigma} = c_{\alpha}, c_{\pi} = c_{\beta} \) and \( i_{\pi} c_{\pi} = [A \cup \text{ci}(A)] \cap \text{ic}(A) \), which are seven in number altogether.
Case 2. Let \((X, \tau)\) be an \(OU\)-space and \(A \subseteq X\). In [2], Aull shows that \(OU\)-spaces are exactly those spaces in which every dense set has dense interior and we know that spaces whose dense sets have dense interiors satisfy ici = ic. For the converse part, let ici = ic for every \(A \subseteq X\) and let \(A\) be a dense set in \(X\). Therefore, \(c(A) = X\) and ic(A) = X. Since ici(A) = ic(A) = X, we have \(X \subseteq ci(A)\). Hence, \(i(A)\) is dense in \(X\). Therefore, in an \(OU\)-space, we have ici = ic and cic = ci. Thus the distinct sets obtained from the operators \(i_\gamma, c_\gamma\) (where \(\gamma = \alpha, \pi, \sigma, \beta\)) are ici = ic, cic = ic, \(i_\alpha = i_\pi, i_\sigma = i_\beta, c_\sigma = c_\beta, c_\pi = c_\alpha\) and \(i_\sigma c_\beta = [A \cap \text{ci}(A)] \cup \text{ic}(A)\), which are seven in number altogether.

Case 3. Let \((X, \tau)\) be a partition-space. Then its open sets are clopen, that is, closed and open simultaneously. Therefore, \(ci(A) = i(A)\) for \(A \subseteq X\). Therefore, in partition-space, we have ici = ic = cic = ic = c. Thus the distinct sets obtained from the operators \(i_\gamma, c_\gamma\) (where \(\gamma = \alpha, \pi, \sigma, \beta\)) are i, c and \(A\) only.

Case 4. Let \((X, \tau)\) be an extremally disconnected \(OU\)-space. Then we have ici = ci = ic = c for \(A \subseteq X\). Therefore, in extremally disconnected \(OU\)-space, the distinct sets obtained from the operators \(i_\gamma, c_\gamma\) (where \(\gamma = \alpha, \pi, \sigma, \beta\)) are ici, \(i_\alpha\) and \(c_\alpha\) only, which are three in number altogether.

Case 5. Let \((X, \tau)\) be a discrete space. Then every singleton is closed and open. Therefore, \(c(A) = i(A) = A\) for \(A \subseteq X\). Therefore, in discrete space, we have only 1 set \(A\) itself, from the operators \(i_\gamma, c_\gamma\) (where \(\gamma = \alpha, \pi, \sigma, \beta\)).

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